VirtuCath™ Software Small Deflection Verification Report

Document ID:	VR-VC-002
Version:	1.0
Date:	March 3, 2026
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1.0 Purpose

The purpose of this document is to provide verification for the VirtuCath™ simulation software. This report formally documents the physical accuracy of the software's core mechanical simulation by benchmarking its performance against established, foundational analytical equations for cantilever beam deflection under a distributed load (gravity).

2.0 Scope

This verification study applies to the core physics engine of VirtuCath. While the software is capable of simulating complex multi-segment, multi-pullwire catheters undergoing active actuation, this study focuses on a simplified, foundational case to specifically verify the underlying continuum mechanics model. The scope is therefore restricted to the quasi-static analysis of a single-segment flexible catheter acting as a cantilever beam under its own weight, without any external loads, tendon actuation, or friction.

This foundational study is a necessary decoupling of variables to isolate and verify the mathematical accuracy of the core multi-body elastic rod solver prior to introducing complex tendon routing forces.

3.0 Referenced Equations

- **1. Transverse Tip Deflection Formulation**

  delta_max = wL^4 / 8EI

  Represents the maximum vertical displacement at the free end of the beam.

- **2. Tip Slope/Angle Formulation**

  theta_max = wL^3 / 6EI

  Represents the maximum rotation (in radians) at the free end of the beam.

4.0 Verification Methodology

The verification process utilizes a randomized approach. An automated pipeline generated randomized permutations of catheter parameters (Length, EI, Linear Density/Mass). For every permutation, the system:

1. Calculates the "Reference" equilibrium state (Maximum Tip Angle and Maximum Deflection) using standard analytical mechanical beam theories.
2. Formats the specific catheter configuration and loads it natively into the VirtuCath SimulationEngine.
3. Applies a sustained lateral 9.81 m/s^2 distributed force (gravity) acting transversally on the catheter backbone.
4. Steps the simulation forward until complete static equilibrium is achieved.
5. Compares the final simulated tip position and angle against the reference values.

4.1 Benchmark Standard: Linear Elastic Cantilever Beam

The benchmark for this study is the analytical equation for a continuous uniform cantilever beam subjected to a constant distributed transverse load (gravity). To maintain validity, the parameter generator explicitly ensures all configurations remain within the linear elastic regime (maximum tip deflection < 5% of total body length).

4.2 Test Configuration

Simulations were performed using a uniform sampling method to generate a valid dataset of 400 permutations for a single-segment, non-actuated catheter configuration.

The parameter space for the generated samples was defined as follows:

- Segment Length (L): 10 mm to 100 mm.
- Catheter Stiffness (EI): 1.0x10^-4 to 1.0x10^-2 N\m^2.
- Linear Density (w/L): 0.001 to 0.02 kg/m

4.3 Comparison Metrics

Two primary outputs were recorded to measure the correlation between the simulation engine and foundational mechanical equations:

Final Tip Deflection (mm): This metric represents the spatial displacement of the distal end. Verifying deflection confirms the overall multi-body discretization effectively represents continuum mechanics.

Final Tip Deflection Angle (deg): This metric represents the terminal slope of the catheter tip position.

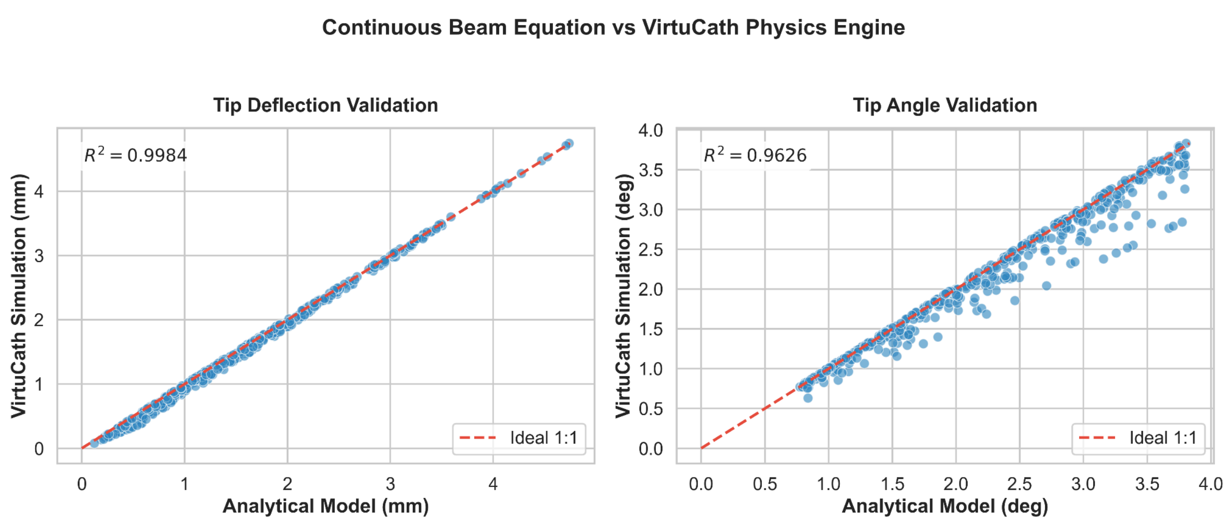
5.0 Results

Deflection Accuracy (mm)

- MAE: 0.0531
- Median Abs: 0.0462
- Standard Dev: 0.0434
- Mean Bias: -0.0515

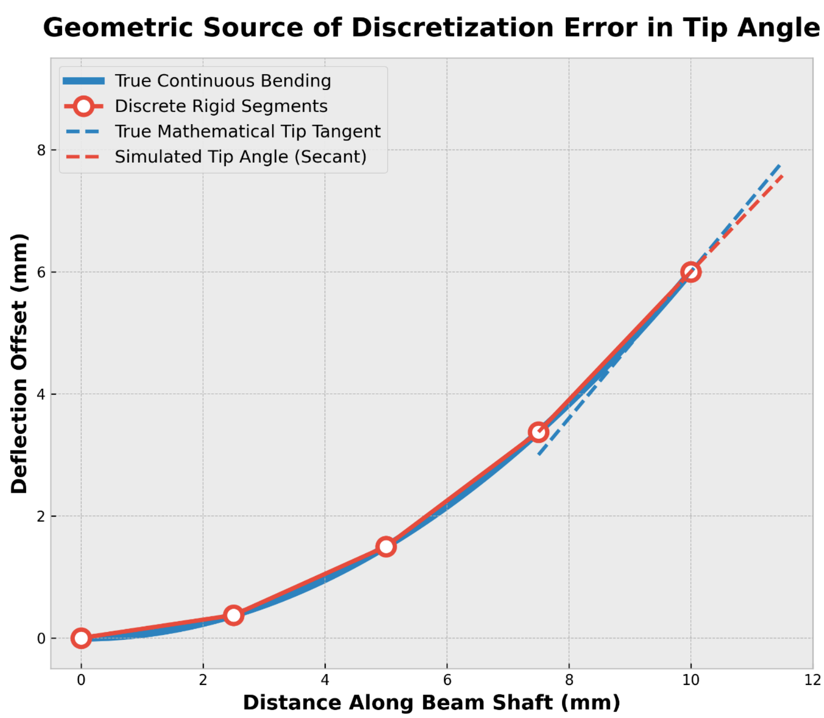
Angle Accuracy (deg)

- MAE: 0.1042
- Median Abs: 0.0366
- Standard Dev: 0.1594
- Mean Bias: -0.0909



6.0 Discretization Effect on Tip Angle

While the tip angle tracking is highly accurate overall, a minor measurable variance exists. This variance strictly correlates with the individual body segment lengths (discretization fidelity). The MuJoCo model builder dynamically allocates rigid bodies to approximate continuous bending. When a very short catheter length hits the minimum body length floor, those individual segments become proportionally large relative to the overall length. This coarser discretization effectively maps fewer tangential approximation nodes, resulting in the scattered tip angle deviations. The absolute variance remains negligible.



7.0 Conclusion

The VirtuCath™ core physics engine is VERIFIED for static cantilever beam deflection under gravity. The model demonstrates high fidelity and negligible bias.

This successful verification provides several critical deductions about the underlying simulation architecture:

Core Mechanics Validity: The engine correctly resolves the fundamental relationship between applied lateral loads, internal restoring moments based on material stiffness EI, and the resulting spatial displacements.

Discretization Fidelity: The rigid-body sequence strategy effectively replicates continuous continuum mechanics. The minor angular variance has been explicitly linked to known geometric secant approximations, confirming that the underlying physical solver is stable and accurate without hidden numerical instability.

Foundational Readiness: Validating this isolated, unactuated cantilever case ensures that the foundational bending physics are mathematically sound. This provides a reliable, verified baseline before introducing complex multi-segment pullwire actuation, variable loading, and contact dynamics in future studies.